TALETE

Teaching mAths through innovative Learning approach and conTEnts

LIFELONG LEARNING PROGRAMME
COMENIUS

Coordinated by Università degli Studi "G.Marconi"

WP5

DEV(21) REPORT ON THE SELECTED ASSESSMENT SHEDULES

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OVERVIEW

This report describes the criteria used by Research Educational Team /RET/ in order to select the international and national schedules on math (geometry) assessment of students' skills. It defines the "mathematisation" concept as the math interpretation of the different real features. It contains the selected schedules: four international schedules and one schedule at national level (of Bulgaria, Greece, Italy and Turkey). The schedules are selected on the base of the following criteria such geometrical field, schedules including more educational objectives, etc. The detailed description of the criteria is given in the document content. On the base of these selected schedules the report describes also the conceptual model (educational training path, social area) to be developed into the TALETE training path. Data and information collected by partners and described in the deliverable provide tailored training contents addressed to the target groups and the comparative framework to policy-makers.

CONTENTS

List of figures

Introduction

The method of open coordination in the EU and the needs for educational reforms in the 21st century call for regular monitoring of educational progress in the member states. At the EU-level several initiatives were taken to promote the use of ICT in primary and secondary education[1].

Next to the expectation that ICT can improve outcomes of learning in traditional subject areas a number of policy documents also mention that ICT can help to implement new ways of learning whereby the students (with the help of ICT) acquire more control and responsibility for their own learning processes and outcomes. For instance digital portfolio's are conceived as a tool that can help to keep track of learning activities and products resulting from these activities.

Based on extensive research, Cisco, Intel and Microsoft concluded that most education systems have not kept pace with the dramatic changes in the economy and the skill sets that are required for students to succeed. These skills include the ability to think critically and creatively; to work cooperatively; and to adapt to the evolving use of technology in business and society.

A curriculum allows governments to regulate (formally and prescriptive or less formally) educational processes in order to influence outcomes of learning. Educational practitioners often mention the time that is needed for realizing the existing curriculum as a major obstacle for implementing ICT in teaching and learning.

For monitoring educational progress at least three main concept areas need to be considered, namely:

- Intended learning outcomes
- Opportunities to Learn (OTL)
- Competencies/attitudes of students.

Definitions of intended outcomes are needed for steering educational processes that result in OTL, which in turn are supposed to influence the competencies and attitudes of students. Moreover these definitions are needed to be able to construct assessment schedules for measuring the extent to which the intentions are realized.

Intentions may be formally legislated in syllabi, examination standards or in the words of IEA_'Intended Curricula'. These constitute the basis for guiding a lot of educational processes, such as the content of the textbooks, teaching and learning activities in schools, the content of (in-service or pre-service) teacher training, etc. An analysis of

[1] The study _Indicators on ICT in Education' was run under the auspices of EACEA (Education, Audiovisual and Cultural Executive Agency of the European Commission). The study is finalized in October 2009.

these intentions is usually the basis for designing international comparative assessments that currently are run by international organizations, such as OECD (PISA) and IEA (TIMSS, PIRLS). These analyses may be based on extensive curriculum analyses (IEA) or expert opinions about what are important life skills that students need to acquire in schools (OECD). The outcomes of such analyses constitute the basis for developing the content specifications for the instruments that are used to measure educational outcomes (e.g. in the cognitive domain, such as mathematics, science and reading, but also affective, e.g. learning motivation), whereas on the other hand these content specifications can also be used for measuring the opportunities that schools offer to students to learn these contents. Educational monitoring that would be only focused on these three core concepts would allow educational actors to make a limited number of inferences, such as:

- At national level:
  - Whether intentions, OTL and outcomes are changing over time
  - Whether discrepancies exist between intentions and OTL
  - Whether inequities exist between sub-populations of students and how these are changing over time.
- At international level:
  - The same as for national level but with enhanced possibilities to interpret the national observations with reference to what is happening in other countries.

TALETE project puts the focus on key areas that will offer promising opportunities for transforming education and assessment.

The main steps that were planned for this study were:

- Identifying concepts from a document analysis and consult national experts (TALETE Project Research Education Team /RET/) in lower and upper secondary schools education about the specification of common criteria for selection of relevant assessment schedules on national and international level in the domain of teaching and learning Math (especially Geometry) for 14-15 years old students;
- Specification of common criteria for selection of international and national schedules. Selected criteria define desirable indicators and search for available indicators and qualify these indicators in terms of measurement quality, feasibility and sustainability;
- Develop a proposal for a set of assessment schedules as a main foundation of the TALETE Prototype development.

RET of educators and academics aims on enabling education assessment methodologies and technologies, effective learning environments, and replicable ICT-enabled teaching and assessment methods that foster the development and assessment of the students' skills. To accelerate the project in time the current versions of PISA and TIMSS as the most broadly used and recognized assessment instruments are taken into account.

Document analysis - International assessment instruments

An international comparative assessment consists of collecting *data* in representative national samples on the basis of *instruments* (usually questionnaires and tests) that contain operationalizations of the intended indicators. There are several issues and constraints that need to be considered when designing an international comparative assessment. First of all, as the instruments are administered to educational actors in schools (school leaders, teachers, students, etc.) a serious constraint is the amount of time that can be asked from each respondent to answer the tests/questionnaires. As the amount of questions that can be included in questionnaires is limited, this in turn has implications for the number of intended indicators that can be included. Initial priority decisions can be made on the basis of a-priori response time estimates. Further, during the process of operationalization and piloting it may appear that the number of intended indicators needs to be further reduced.

Trends in International Mathematics and Science Study /TIMSS/ and Program for International Student Assessment /PISA/ are international instruments. The primary aim of these assessments is to assess different features of student learning.

“While national monitoring provides evidence regarding educational progress in one country, often countries feel the need for international benchmarks for better interpretation of the national educational developments. In recent decades the interest for and participation in international comparative educational monitoring has increased substantially as witnessed by the ever growing number of countries that participate in the international comparative educational assessments”[2].

These assessments are intended to assist policy makers to better understand to what extent their educational systems are measuring up with developments taking place in other countries. Since the mid 1980s many governments have made major investments to equip schools with modern technologies in order to modernize teaching and learning and to provide students with opportunities to learn about these technologies and to acquire competencies that they will need in their future life.

[2] The study ‘Indicators on ICT in Education’ was run under the auspices of EACEA (Education, Audiovisual and Cultural Executive Agency of the European Commission), page 122.

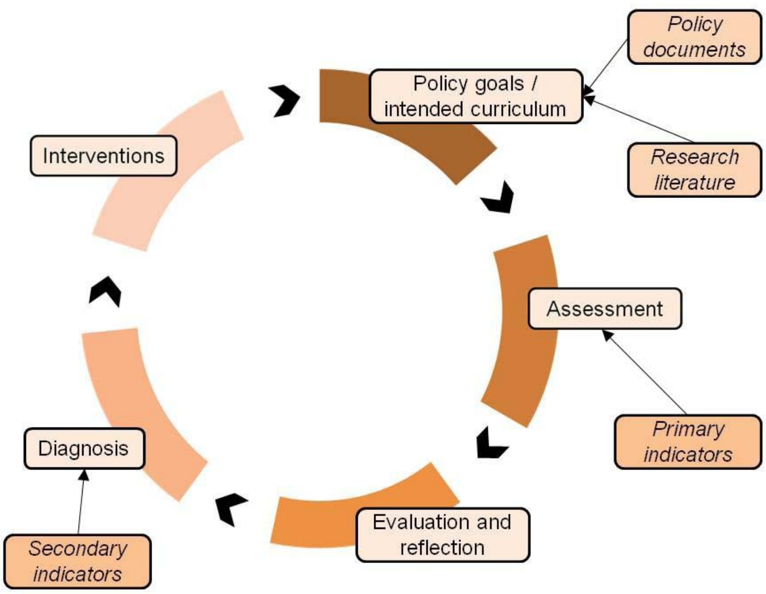


Figure 1. Main steps in policy cycle

Figure 1. Concerns a very general model that can be applied in many different settings, for instance at the international (worldwide, regional), national, school and even individual level. Given the purpose of our study we will further focus mainly on the international level and will describe below in more detail each of the steps that are distinguished in Figure 1. and in particular in terms of what is required in each of these steps, which concepts (shown in *italics*) are relevant, and which questions and dilemmas will be faced. Monitoring implies a regular repeat of step 2.

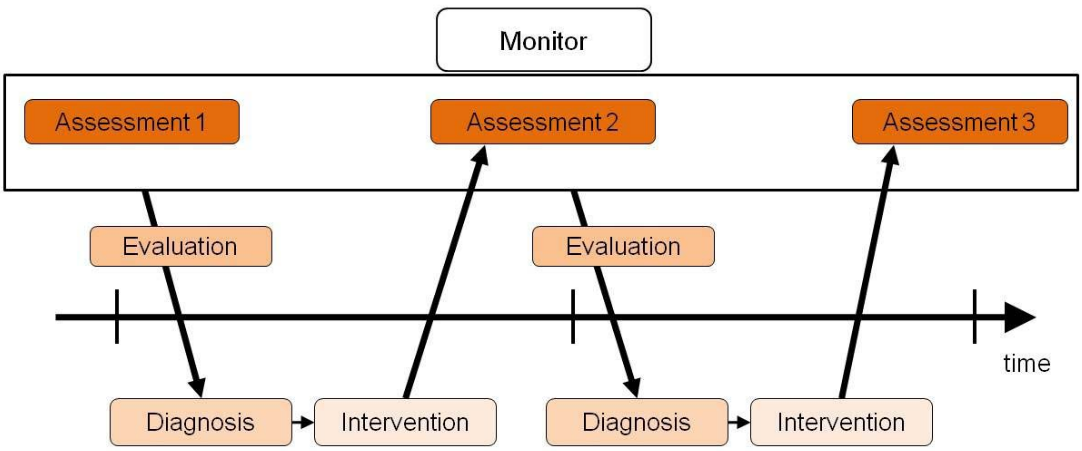


Figure 2. Monitoring as a regular repeat of assessments

An important distinction in Figure 1. is between *primary* and *secondary indicators* (sometimes also called respectively key indicators and background or explanatory indicators). Primary indicators are those that are featured as the main focus of an assessment, for instance when it concerns PISA or IEA-TIMSS-PIRLS primary indicators concern the test results in mathematics, science and/or reading, which are usually the first to be featured when statistical reports from these international monitors are released. Secondary indicators are used to throw further light on the test results, for instance by examining difference in outcomes between sub-populations in countries (e.g. boys and girls) or for analyzing how the differences between countries can be explained. The distinction between primary and secondary indicators points to the dual role of indicators. First for identifying problems (via the primary indicators) and secondly for finding potential causes for the existence of problems and providing indications for developing policies aimed at educational improvements.

The intentional differences in the projects are evident in how each project structured and developed its tests.

In general terms, TIMSS sought to find 'what students know' and PISA sought to find 'what students can do with their knowledge'. These two perspectives are neither ‘better’ nor ‘worse’. Rather they are different, and each has importance as a learning outcome of the study of science at school.

The data gathered in the TIMSS project related to the intended curriculum (the curriculum specified by the system or other body), the implemented curriculum (the curriculum as taught by teachers, the nature of actual classrooms), and the attained curriculum (what students have learned).

The PISA project is not directly focused on any of these aspects of curricula. Rather PISA is concerned with how well 15 year old students can make use of science knowledge acquired from school and from other sources, in situations in everyday life that involve science and technology.

Trends in International Mathematics and Science Study /TIMSS/

TIMSS gathered data from samples of the student population at three levels (middle primary, lower secondary, final year secondary), from the teachers of these students, and from their schools and systems.

Development of the tests of student learning outcomes for each student population began with an analysis of science curriculum guides and textbooks from many countries to 'identify priority topics' for the tests. An international panel of science curriculum specialists then produced a framework to guide test development.

Framework dimensions

The framework has the following dimensions:

- *a content dimension* which indicated the proportions of test questions required for each of the areas of science (life science, earth science, physical science, etc.), and
- *a performance expectations* dimension for what was likely to be involved in answering the items (understanding simple information, solving problems, using science processes, etc.).

There was also a *perspectives dimension* that included science-related details about the individual student and his/her classroom and school contexts (attitudes, interests, habits of mind, and so on).

Tests for different levels had different proportions of items on the content and performance expectations dimensions, and included multiple choice, short answer and free response items.

The free response items usually required students to answer a question and then explain their answer. On a much smaller scale, some of the students from the first two populations undertook a 'Practical Performance Test' that required them to carry out a range of tasks and experiments involving simple scientific equipment. No comments are made about this performance testing in this digest as no international results have been published.

A School Questionnaire was used to gather information about the intended and implemented curriculum, and a range of school characteristics (location, size, resources, curriculum offerings, etc.).

A Teacher Questionnaire asked about qualifications, levels taught, approaches to planning and carrying out teaching, use of textbooks and other resources, views on current curricular issues, etc.

A Student Questionnaire sought information about demographic details, how students spent time, attitudes to science, expectations, etc. Finally, data for curriculum analyses were collected at the system-level, together with details about structural aspects that differed across the countries[3].

Performance expectations dimension

- Understanding simple information
- Understanding complex information
- Theorizing, analyzing, solving problems
- Using tools, routine & science processes
- Investigating the natural world

[3] TIMSS questionnaires have been published on the following address: http://nces.ed.gov/timss/questionnaire.asp

Program for International Student Assessment /PISA/

PISA put the emphasis on mathematical literacy and ‘preparation for life’. The beginning point for the testing is quite different to TIMSS. After consideration by the science expert group, the following definition of mathematical literacy was adopted by PISA for its testing:

“Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen."

This definition includes mathematical thinking and use of mathematical concepts, procedures, facts and tools for describing, explaining and statement of hypotheses about processes and phenomena. In other words, in mathematics PISA assesses students' skills to formulate, use and interpret mathematical problems in a variety of situations.

The following figure represents the mathematical literacy elements according PISA.

Mathematisation process

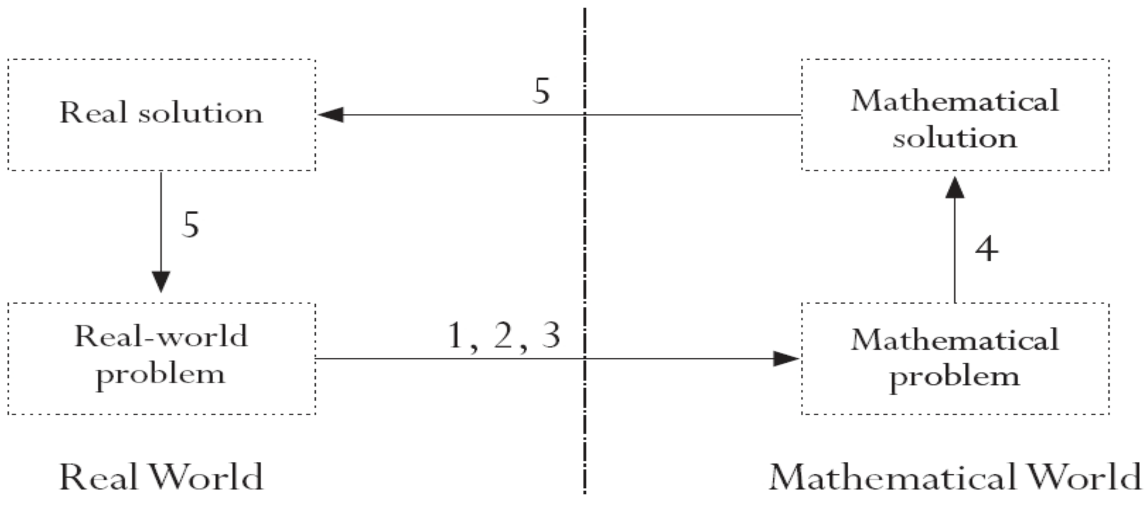


Figure 3. Mathematisation process according PISA

Stage1
1. Starting with a problem situated in reality
2. Organising in according to mathematical concepts
3. Taking into account the mathematical features of the situation and transforming real-life problem into mathematical problem

Stage 2
4. Solving the mathematical problem

Stage 3
5. Making sense of the mathematical solution in terms of the real situation, including the identifying the limitations of the situation

Mathematisation first involves translating the problem from “reality” into mathematics. This stage of the process includes activities such as:

- Identifying the relevant mathematics with respect to a problem situated in reality;
- Representing the problem in a different way; including organization it according to mathematical concept and making appropriate assumptions;
- Understanding the relationship between the language of the problem, and symbolic and formal language needed to understanding mathematically;
- Finding regularities, relations and patterns;
- Recognizing aspects that are isomorphic with known problems;
- Translating the problem into mathematics; *i.e.* to a mathematical model, (de Lange, 1987, p. 43)

After translating the problem into a mathematical form, the next stage includes:

- Using and switching between different representations;
- Using symbolic, formal and technical language and operations;
- Refining and adjusting mathematical models;
- Combining and integrating models;
- Argumentation;
- Generalization.

The last step(s) in solving a problem involve reflecting of the whole mathematisation process and the results.

- Understanding the extend and limits of mathematical concepts;
- Reflecting on mathematical arguments, and explaining and justifying results;
- Communicating the process and solution;
- Critiquing the model and its limits.

Mathematical literacy is assessed in the context of:

Mathematical content

There are four distinct content areas: quantities (including the use of numbers for quantitative characteristics and relations between real objects); space and forms (recognition of shapes in different configurations and sizes, looking for similarities and differences in the analysis of figures and their elements, knowledge of the properties of objects and their mutual disposition) functions and relations (mathematical description of different processes) probabilities and data (working with data, graphical presentation and interpretation).

Mathematical competencies

An individual who is to engage successfully in mathematisation within a variety of situations, extra- and intra-mathematical context, and overarching ideas, needs to posses a number of mathematical competencies which, taken together, can be seen as constituting comprehensive mathematical competence. To identify and examine these competencies PISA make use of eight characteristics:

- Thinking and reasoning;
- Argumentation;
- Communication;
- Modeling;
- Problem posing and solving;
- Representation;
- Using symbolic, formal and technical language and operations;
- Use of aids and tools.

These characteristics define the main competency clusters of the mathematical competencies: *reproduction (*performing simple mathematical operations); *determine the logical relationships* (generalization of ideas to solve a problem); *reflection* (demonstration of mathematical thinking and reasoning).

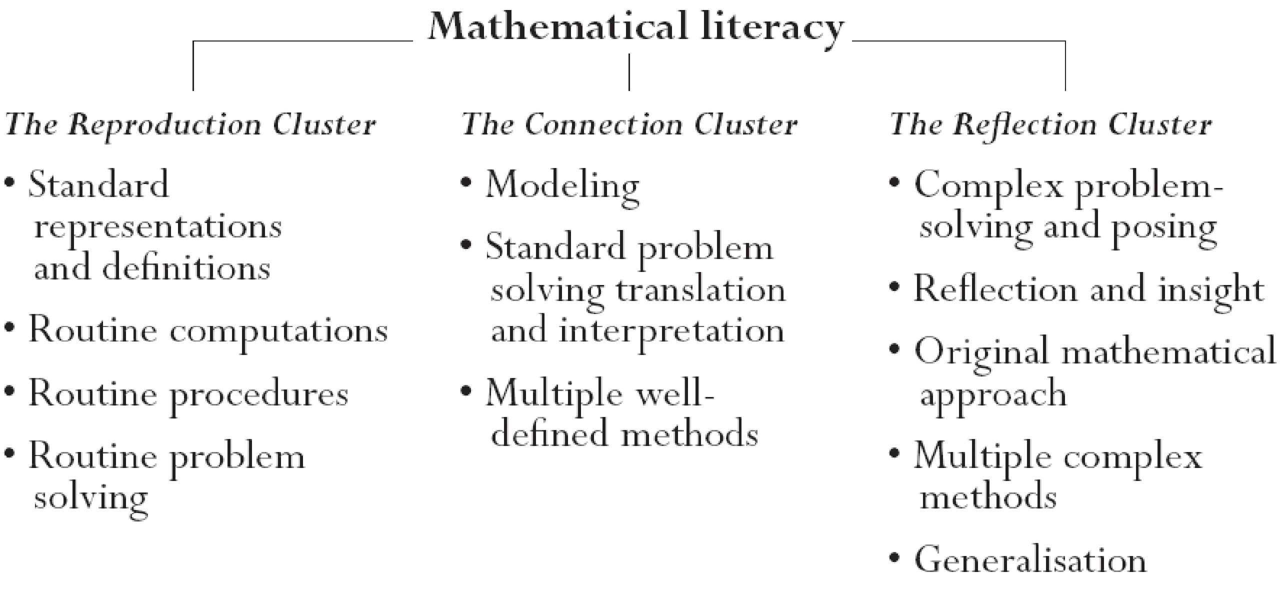


Figure 4. Competency clusters according PISA

The figure above represents the main competencies clusters and distinctions between them.

- **Measurement of cognitive processes:** formulate, use, interpret.

Thus described a conceptual framework for assessing mathematical literacy in PISA is summarized in the table:

	Mathematical literacy
Definition and distinctive characteristics	The ability of students to formulate, interpret and use mathematical knowledge in a variety of situations. It includes mathematical thinking and use of mathematical concepts, procedures, facts and tools for describing, explaining and Hypothesizing about processes and phenomena. Allows students to conceptualize the role of mathematics in the modern world, formulate reasoned arguments and to use mathematical knowledge in a way that meets the needs of today's active and constructive citizen.
Content	Content domains and concepts: • Quantities • Space and forms • Functions and relations • Probabilities and data.
Competencies	Competence is measured by the necessary mathematical skills: • Reproduction • Determination of logical dependencies/connections • Consideration.
Cognitive processes	• Formulates • Uses • Interprets.
Context	Areas of application of mathematics with an accent on its use in the following aspects: • Individual • Educational and professional • Social • Scientific

National curriculums analysis

This study covers also an analysis of national educational standards in mathematics described in national curriculums of all the countries involved in the educational research process of the TALETE Project – Bulgaria /BG/, Greece /EL/, Italy /IT/, and Turkey/TR/. This step is very important in terms of the future use of the developed under the project products in secondary schools on regular basis. This chapter represents the main results of the national curriculums of Math for 14-15 years old students.

The results of the curriculums analysis help the TALETE RET to determine what are the common aspects and objectives of the training in Math (especially Geometry) on international level. The identified common objectives could be summarized as follows:

1. Learning the concept of vector in the plane of the affine operations with vectors and their applications.
2. Assimilation of the uniformities in the plane.
3. Deepen and broaden students' knowledge of geometry figures by studying the mutual positions of the circumferences, of a circumference and angle, of a circumference and polygon, and properties of the remarkable points in a triangle.
4. Deepening of knowledge and logic skills, formation of logical culture and learning of mathematical language.
5. Assimilation the basic applications of the mathematical knowledge, showing integrative functions of mathematics.
6. Forming a positive attitude towards mathematics, creating interest and motivate students to acquire knowledge and skills.
7. Development of observation, imagination, concentration of the thinking, memory.
8. Mastering of objective criteria for evaluating the spiritual and material values of the society.
9. Building habits to protect the environment and personal health.

In order to select assessment schedules on international level TALETE RET needed common framework defining the indicators and attributes for the description of the Geometry topics covered by national educational curriculums. The selected set of attributes allows RET to analyze specifics of the national standards but also to render an account the compliance with the indicators of TIMSS and PISA programs.

Last but not least the development of common framework for Geometry topics description helps the technological team in the process of decision making concerning the prototype of the 3D educational environment development in terms of conceptual model, design, and functionality.

Concerning the Geometry topics (schedules) representation by RET were selected two main classes of attributes:

- Expected results – this class of attributes covers the following:
  - Description of nucleus of the learning content
  - Description of the expected results at a curriculum level
- Learning content – this class of attributes describes the separate topics included in given curriculum nucleus, the basic terms and main concepts students have to learn, the context and activities and the possibilities for representation of inter-subject relations. Thus the topics' content is determined on the basis of:
  - Standards that students must meet as a result of completion of the appropriate level of secondary school;
  - Outcomes that students must achieve;
  - Opportunities that provide the curriculum;
  - Link the subject of mathematics with its objects and other cultural and educational fields.

On the basis of selected attributes RET developed common template for description of the Geometry topics covered by national curriculums of the partner countries.
The template structure is presented on the figure below.

Expected results		Learning content (topics, concepts, context and activities, inte-subject relations)			
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*	*Possibilities for inter-subject relations*

Figure 5. Attributes for Geometry topics (schedules) description

The descriptions provided by partners are presented in APPENDIX 1 of this document.

Common criteria for selection of assessment schedules

Specification of common criteria for selection of international and national schedules is based on the results of both the international assessment instruments and schemes more concretely TIMSS and PISA and the national educational curriculums of the countries involved. The selection of assessment schedules was based of unified descriptions of national educational curriculums. Provision of such unified set of attributes allow RET to select appropriate and relevant assessment schedules which are in accordance of the national standards but last but not least cover the aims and criteria of the international instruments TIMSS and PISA.

Procedure of selection

The procedure for selection of international and national assessment schedules had several stages:

Stage1: Development of the template for unified description of the Geometry topics covered by educational curriculums of the partner countries.

The technological team selects the topic and schedules for the development of the TALETE Prototype.

Stage2: Each partner country provides 4 to 6 assessment schedules which are:

- Relevant to the described Geometry topics in terms of:
    - Standards that students must meet as a result of completion of the appropriate level of secondary school;
    - Outcomes that students must achieve;
    - Opportunities that provide the curriculum.
- The provided assessment schedules must be appropriate for representation in 3D interactive educational environment.
- Assessment schedules must allow the implementation of PISA Matematisation process and its stages.
- Accordance with PISA Competency Clusters and their concrete semantics in terms of thinking and reasoning; argumentation; communication; modeling; problem posing and solving; representation; using symbolic, formal and technical language and operations; use of aids and tools[4].

Stage3: RET of each partner country revises the proposed assessment schedules in respect to the criteria described above.

Stage4: RET of each partner country votes each of the proposed schedules in order to select these which are appropriate to be realized into the TALETE environment on

[4] OECD, The PISA 2003 Mathematical Literacy, pp. 40-49, http://www.pisa.oecd.org/dataoecd/38/51/33707192.pdf

international level and these which are appropriate to be realized on national level. The voting results are sent to the package leader in order to be summarized.

Stage5: The WP5 package leader (BFU, Bulgaria) summarizes the received results and finalized the proposal for the assessment schedules which will be took into account further. International schedules are four (one from the proposals of each country). They are these of the schedules which are with maximum votes. The national schedules are selected also taking into account international voting. The reason is that these schedules are proposed by corresponding national RET which means they are in accordance with the national curriculum. From the other side, these schedules are voted also by the international RET which means that they could be useful and appropriate as didactic materials in other countries too.

The results from stages 3 and 4 of the selection process are briefly presented in the table bellow:

BULGARIA	**International**
	1. Bulgaria - exercise 3 - cakes
	2. Bulgaria - exercise 4 - swimming pools
	3. Greece - 3 - paralel lines
	4. Greece - 1 - similar triangles
	5. Italy - exercise 1 - roman mosaic
	6. Italy - exercise 2 - separation wall
	7. Turkey - exercise 4 - polygons&steps
	8. Turkey - exercise 3 - rectangle prism
	National :
	1. Bulgaria - exercise 5 - bicycles
	2. Bulgaria - exercise 6 - cogwheel
GREECE	**International**
	1. Greece - similar triangles
	2. Bulgaria - exercise 4 - swimming pools
	3. Turkey - exercise 1 - triangles rotation
	4. Italy - exercise 2 - separation wall
	5. Turkey - exercise 2 - calendar pages
	6. Italy - exercise 1 - roman mosaic
	7. Bulgaria - exercise 3 - cakes
	8. Greece - paralel lines/Thalès
	National :
	1.Greece - similarity
	2.Greece - homothety
ITALY	**International**
	1. Bulgaria - exercise 1. - windmill
	2. Bulgaria - exercise 2. - carnival hats
	3. Bulgaria - exercise 4. - swimming pool
	4. Greece - similar Triangles
	5. Greece - similarity
	6. Turkey - excerise 1 - triangles rotation
	7. Turkey - excerise 2 - calendar pages
	8. Turkey - excerise 3 - rectangle prism
	National :
	1. Italy- excercise n.1 - roman mosaic
	2. Italy - excercise n.3 - cube
TURKEY	**International**
	1. Bulgaria - exercise 3 - cakes
	2. Bulgaria - exercise 4 - swimming pools
	3. Greece – homothety
	4. Greece – similarity
	5. Italy - exercise 1 - roman mosaic
	6. Italy - exercise 3 - cube
	7. Turkey - exercise 1 - triangles rotation
	8. Turkey - exercise 2 - calendar pages
	National :
	1. Turkey - exercise 3 - rectangle prism
	2. Turkey – exercise 4 - polygons&steps

Table 1. Results from voting on national level.

Next table presents the final results from the process of summarization of the votes.

International Selection		voting BG	voting EL	voting IT	voting TR	TOTAL
Bulgaria RET	windmill			1		1
	carnaval hats			1		1
	cakes	1	1		1	3
	swimming pools	1	1	1	1	4
Greece RET	paralel lines / Thales	1	1			2
	similar triangles	1	1	1		3
	similarity			1	1	2
	homotethy				1	1
Italy RET	Roman mosaic	1	1		1	3
	separation wall	1	1			2
	cube				1	1
	pot					0
Turkey RET	triangles rotation		1	1	1	3
	calendar pages		1	1	1	3
	rectangle prism	1		1		2
	polygons&steps	1				1

Legend:
- national schedule (light blue)
- international schedule (yellow)

Table 2 Summarized results from International TALETE RET voting

As is visible the national assessment schedules with highest voting results are selected and will be realized on international level. The schedules which are on second place in the national group will be realized on national level. Where the voting results are equal the final decision is took by extra discussing with the experts.

APPENDIX 1 – Unified Geometry topics descriptions

Geometry topics selected by Bulgarian RET

Expected results		Learning content (topics, concepts, context and activities, inter-subject relations)		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Figures and Bodies	**Standard 1:** The student knows the basic geometry figures (triangle, quadrangle), their elements and properties. **Expected result:** The student knows and properly uses the properties of midpoint section in a triangle and trapezium and centroid of a triangle.	**Topic. Midpoint section** The student knows: 1. what " midpoint section in a triangle" is, its properties, and knows how to use them; 2. what " midpoint section (base) of a triangle" is, its properties, and is able to apply them when necessary; 3. what „a centroid of a triangle" is, its properties, and is able to apply them when necessary; 4. how to find and make situations related to midpoint sections.	midpoint section of a triangle; midpoint section in a trapezium; centroid of a triangle.	The students should be enabled to: - become familiar with the different ways of proving the properties of the midpoint section and centroid (with vectors, congruent triangles, areas, etc.).
Logical knowledge	**Standard 1:** The student knows, at a specific level, the meaning of the logical linkers „and“, „or“, „if..., then“ and of the relation equivalence.	1 The student is able to distinguish the statements out of the topic as necessary and sufficient terms; 2. The student is able to form a negation of statements, that are related to the topic by content;		

	Standard 2: The student knows, at a specific level, how to form a negation of a statement, having the logical linkers „and“/„or“ in it. **Standard 3:** The student is able to estimate what suitable and rational is in a specific situation. **Expected result:** The student is able to give proofs on the base of the logic structure of the studied theory.	3. The student is able to analyze the meaning of a specific statement, and to choose appropriate means as a proof.		
Expected results		**Learning content (topics, concepts, context and activities, inter-subject relations)**		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Modeling	**Standard 1**: The student knows the meaning of the term “vector”, how to add and subtract vectors, to multiply a vector by a figure. **Expected result:** The student knows how to make afine operations with vectors and use them.	**Topic: Vectors** 1. The student knows the meaning of the term “a vector” and the terms related to it; 2. The student knows different operations with vectors, their properties, is able to make operations and apply them; 3. The student is able to present a vector, in a specific situation, as a linear combination of vectors.	same-direction rays; rays with opposite directions; direction; line; directed section; vector; zero vector; length of a vector; direction of a vector; same-direction vectors; vectors with opposite direction and equal lengths; congruent vectors;	The students should be enabled to: - become familiar with basic vector equations; - become familiar with the different ways of proving the properties of midpoint section and centroid with vectors - use the vectors

			vectors with opposite direction and equal lengths; sum of vectors; difference of vectors; product of a vector with a figure; collinear vectors	as means of proving equations and parallelism of sections, coincidence of points, collinearity of points - become familiar with the basic application of the vectors in Physics.
Expected results		**Learning content (topics, concepts, context and activities, inter-subject relations)**		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Functions. Measuring	**Standard 1**: The student knows how to make an image of a point, section and circumference at congruency. **Expected result:** The student makes images of familiar geometry figures at congruency.	**Topic. Congruency** The student: 1. knows what geometry transformation and congruency transformation in the plane are; 2. knows what the different types of planes are as well as the terms related to them; 3. is able to construct an image of a point, section and circle at congruency.	Geometry transformation; congruency; image; prototype; axis symmetry; axis of symmetry; symmetrical points; central symmetry; center of symmetry; rotation; oriented angle; center of rotation; translation; vector translation.	The students should be enabled to: - construct images of geometry figures at congruency, or composition of congruencies; - relate their knowledge of congruencies to graphs of functions.
Expected results		**Learning content (topics, concepts, context and activities, inter-subject**		

		relations)		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Figures and Bodies	**Standard 1:** The student knows the basic geometry figures (triangle, quadrangle, regular polygon and circumference), their elements, types and characteristics. **Standard 2:** The student knows to construct geometry objects, described in the basic construction tasks. **Expected result:** The student knows the allegations about inscribed and circumscribed polygons and knows how to use them.	**Topic - Circumference And Polygon** The student: 2. knows and is able to determine the reciprocal positions of: • points and circumference; • straight line and circumference; • two circumferences; 3. knows and is able to apply the features of chords into a circumference; 4. knows the distinctive points of a triangle and statements, related to it; 5. knows the geometric place of points, from which a certain segment is seen under a certain angle, and is able to construct it; 6. relates familiar geometric object to the term 'geometric set of points' and uses them in tasks of constructional matter; 7. knows the necessary terms for inscribed and circumscribed tetragons, and is able to apply them; 8. knows to construct a tangent from an outside point to a circumference.	• Geometric set / place of points; • Internal point for a circumference; • External point for a circumference; • Tangent to a circumference; • Tangent point; • Secant of a circumference; • External-tangent circumferences; • Internal-tangent circumferences; • Concurring circumferences; • Concentric circumferences; • Center of two circumferences; • Joint tangent to two circumferences; • Circumscribed circumference around a polygon;	The students: 1. can make a triangle out of different combinations of certain elements; 2. are able to understand some geometric places of points and their applications; 3. construct joint tangent to two circumference s

			• Inscribed polygon; • Inscribed circumference in a polygon; • Circumscribed polygon; • Orthocenter; • Center of a circumscribed circumference for a triangle; • Center of the inscribed circumference for a triangle	
Functions. Measuring	**Standard 1:** The student knows how to specify in terms of type, and to recognize angles related to a circumference. **Expected result:** The student uses his/her knowledge on measures of angles, relating to a circumference in specific geometric situations. .	The student knows what the different types of angles relating to circumference are, the statements about them, and knows how to apply them.	• Catenary pertaining to a central angle; • Inscribed angle; • Peripheral angle; • Angle whose top is an internal point for a circumference; • Angle whose top is an external top for a circumference	
Logical knowledge	**Standard 1:** The student knows at a specific level the point of the relation Equivalence. **Standard 2:** The student knows the meaning of the words: Theorem property and	The student: 1. knows and applies the indications and properties for an inscribed and circumscribed quadrangle; 2. knows how to differentiate and create situations into which he / she applies Theorems property and Theorems		

	Theorem indication **Standard 3:** The student knows to estimate the accuracy and rationality in a specific situation. **Expected result:** The student finds and uses the logical structure of statements	indication; 3. knows how to formulate hypothesis and to verify it; 4. understands the meaning of the relation Equivalence and can ground the equivalence of statements.		

Geometry topics selected by Greece RET

Expected results		Learning content (topics, concepts, context and activities, inte-subject relations)		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Knowing	**Expected result:** Be able to formulate the theorem of Thales and write the corresponding proportions in any shape Know the conditions of the theorem.	**Topic - Thales Theorem** The student: 1. knows and is able to recognize a Thales shape in a trapezoid 2. knows and is able to recognize a Thales shape in a triangle 3. knows and is able to recognize a Thales shape in a interconnected triangles	• Parallel lines • Intersections • Triangles • Trapezoids • Fraction • Ratio	The students: 1. can apply the theorem in a general trapezoid form 2. can apply the theorem in a single triangle 3. can apply the theorem on several interconnecting triangles 4. understand that the theorem can apply also when the non-parallel lines intersect with the parallel ones
Applying	**Expected result:** Learn to use the theorem of Thales to calculate the length of a segment and	The student: 1. is able to compute a fraction representing a ratio 2. is able to place correctly points		

Reasoning	therefore two parts. **Expected result:** Learn to use the converse of the theorem of Thales to prove parallelism	on a graphic so that their position complies with a predetermined ratio The student knows that if the ratio between two segments is equal then they are parallel Conversely, the student knows that if the ratio between two segments is not equal then they are not parallel		
Expected results		**Learning content (topics, concepts, context and activities, inte-subject relations)**		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Knowing	**Expected result:** Know Euclid's fifth postulate Know the consequences of this postulate on triangles and quadrilaterals.	**Topic - Parallel lines** The students knows that if a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles. The students knows that if through the middle of the side of the triangle a line parallel to the base of the triangle is drawn, then this line crosses the other side of the triangle in its middle The students knows that if through the middle of the side of a quadrilateral, a line parallel to the base of the	Parallel Lines Segments Ratio	The student will: Know that the length of segments formed by the intersection of a line with parallel lines is equal Know that in a triangle (and a quadrilateral), the intersection of a line parallel to a side and going through the opposite side will cut the base side in its middle Deduce the

Applying **Reasoning**	Know how to use common tools in figures Know what is called a ratio of segments and how it is calculated Know when two segments are similar to two other segments	quadrilateral is drawn, then this line crosses the other side of the quadrilateral in its middle Know how to divide with a ruler and a compass a segment in n equals parts Know how to compute a ratio by dividing the length of a segment by the length of another segment Know when two segments are similar to two other segments because their ratio is proportional		length of the segment formed by the line parallel to the base of the triangle (quadrilateral) and going through the middle of a side Know how to use a ruler and a compass to create and measure segments Know how to use fractions to solve problems with parallel lines
	Expected results	**Learning content (topics, concepts, context and activities, inte-subject relations)**		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Knowing	**Expected result:** Know the types of triangles and important characteristics of a triangle (primary and secondary).	**Topic - Similar triangles** The student: 1. knows and is able to recognize the altitudes and orthocenter of a triangle 2. knows and is able to recognize the bisectors of a triangle 3. knows and is able to recognize the medians of a triangle 4. knows and is able to recognize	• Angles • Edges • Sides • Altitudes • Bisectors • Medians • Vertices • Equilateral • Isosceles	The students: 1. recognize triangle shapes 2. Are familiar with the lines associated to a triangle

		the vertices of a triangle 5. knows and is able to recognize an equilateral triangle 6. knows and is able to recognize an isosceles triangle 7. knows and is able to recognize a scalene triangle 8. knows and is able to recognize an acute-angled triangle 9. knows and is able to recognize an obtuse-angled triangle 10. knows and is able to recognize a right triangle	• Scalene • Acute-angled • Right • Obtuse-angled	
	To learn the three criteria that defines similarity of triangles.	• *AA*: if two triangles have two corresponding pairs of angles with the same measure then they are similar. Sometimes this criterion is also referred to as *AAA* because equality across triangles of two angles implies equality of the third. This criterion means that if a triangle is copied to preserve the shape, then the copy is to scale. • *SSS* (*Three sides proportional*): If the ratio of corresponding sides of two triangles does not depend on the pair of corresponding sides chosen, then the triangles are similar. This means that any triangle copied to scale is also copied in shape. • *SAS* (*Ratio of two sides, included*		The students: can apply the three criteria to any non-right triangles in order to determine if two triangles are similar

	To learn the criteria similarity of right triangles.	*angle*): if two sides in a triangle are proportional to two corresponding sides in another triangle, and the angles included between these sides have the same measure in each triangle, then the triangles are similar. This means that to enlarge a triangle, it is sufficient to copy one angle, and scale just the two sides that form the angle. The student knows that two right triangles are similar if: • if one of the acute angles of the first is equal to one of the acute angles of the second • one side and one acute angle are equal The student knows the criteria of similarity for all types of triangles and knows how to apply them.		The students: can apply the two criteria to any right triangles in order to determine if two triangles are similar With the help of transparent paper the students will see from the ABC triangle coincides with the triangle DEF. So they find out that the corresponding sides and angles of triangles are equal and will result in the wording of the definition of equal triangles.
Applying	**Expected result:** The student will be able to apply the criteria to prove the equality of segments and angles.			
Reasoning	**Expected result:** Understand when two			

	triangles are equal, if necessary with the appropriate shift to coincide with another	Students will be able to recognize quickly if two triangles are similar and know how to prove it		
Expected results		**Learning content (topics, concepts, context and activities, inte-subject relations)**		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Knowing	**Expected result:** Know that two similar polygons are a scaled up or down version of each other. Be aware that if two polygons have their sides proportional and corresponding angles equal, then they are similar and vice versa.	**Topic - Similarity** The student: • Knows that two geometrical objects are called similar if they both have the same shape • Knows that either one is congruent to the result of a uniform scaling (enlarging or shrinking) of the other. • Knows that this means that either object can be rescaled and repositioned so as to coincide precisely with the other object. • Knows that corresponding sides of similar polygons are in proportion, and corresponding angles of similar polygons have the same measure. • Knows that one can be obtained	Similarity	The student knows how to recognize similar figures from a panel of examples The students knows the difference of the mathematical strict definition of similarity and the more loose common use of the term on the day to day language The student knows how to fill in some table with length of segments and angles value to verify if several

Applying	Know what the ratio of a similarity between polygons is and what relationship it has with the ratio of their circumferences. Knowing what is the ratio of similarity in real life.	from the other by uniformly "stretching" the same amount on all directions, possibly with additional rotation and reflection • Knows that all the sides of a similar figure are multiplied by the same value compared to the original • Know that it's a multiplication which govern the ration between the sides of two similar figures and not an addition (very common mistake) • Know how to apply his knowledge of similarity to solve practical exercises with fields or floor plans		figures are similar The student knows how to compute angles values and side length of two polygons which are defined as similar but where all the data is not known The student knows how to determine if polygons are similar given a figure with incomplete data about their side length and angle values The student knows how to recognize the case of similarity in practical exercises and how to use his knowledge of the lesson to find practical answers.

Geometry topics selected by Italian RET

Expected results		Learning content (topics, concepts, context and activities, inte-subject relations)		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Number	**Standard 1:** One knows all the fundamentals operations with numbers. **Expected result:** One knows to compute with integers, fractions and decimals, be able to solving problems using mathematical tools.	**Topic - Operations and Numbers** The student: 1. Knows and masters the specific contents of Mathematics (mathematical objects such numbers and their specific operations); 2. Knows and masters algorithms and procedures (most of all in arithmetic); 3. Is able to acquire progressively typical forms of mathematical thought (surmise, verify, warrant, define, generalize).	Computing, estimating or approximating with whole numbers. Representing decimals and fractions using words, numbers, or models (including number lines) Computing with fractions and decimals.	The students: 3. Are able to understand the fundamental s of numerical operations 4. Are able to solving problems involving numbers and their properties
Algebra	**Standard 1:** One knows the basic of the algebraic operations. **Standard 2:** One knows how to solve linear equations using formulas.	**Topic - Relations and Functions** The student: 1. Works with sums, products, and powers of expressions containing variables; 2. Is able to evaluate expressions for given numeric value;	Representing, comparing, ordering, and computing with integers. Problem solving involving per cents and proportions.	The students: 1. Are able to understand how to solve expressions

	Expected result: To learn solving problems using the tools of Mathematics (locate and connect relevant information, compare solution strategies, identify issues predicting patterns such as sequence of tasks, exposing the resolution procedure).	3. Is able to simplify or compare algebraic expressions; 4. Is able to model situations using expressions; 5. Evaluates functions/formulas for given values of the variables; 6. Solves simple linear equations and inequalities, and simultaneous (two variables) equations.	Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalization of patterns. Simplifying and evaluating the algebraic expressions. Simple linear equations and inequalities, and simultaneous (two variables) equations. Equivalent representations of functions as ordered pairs, tables, graphs, words, or equations. Modeling situations using expressions.	containing variables 2. Are able to model situations using expressions
Geometry	**Standard 1:** One knows to recognize shapes in space (recognizing shapes in different representations, identify relationships between shapes, images, or Visual representations, visualize	**Topic - Figures and Bodies** The student: 1. Knows the relationships for angles at a point, angles on a line, vertically opposite angles, angles associated with a transversal cutting parallel lines,	Geometric properties of angles and geometric shapes (triangles, quadrilaterals, and other common polygons).	The students: 1. Are able to construct and interpret different geometric

	three-dimensional objects from two-dimensional representation, and conversely, represent a solid plan, seizing the properties of objects and their relative positions). **Expected result:** To know and master different forms of representation and know how to go from one to another (verbal, written, graphical, symbolic);	and perpendicularity 2. Knows the properties of geometric shapes: triangles, quadrilaterals, and other common polygons 3. Recognizes congruent figures (triangles, quadrilaterals) and their corresponding measures 4. Recognizes similar triangles and recall their properties 5. Knows the relationships between two-dimensional and three-dimensional shapes 6. Knows how to measure, draw, and estimate the size of angles, the lengths of lines, areas, and volumes 7. Knows formulas for perimeters, circumferences, areas of circles, surface areas, and volumes 8. Knows the measures of irregular or compound areas (e.g., by covering with grids or dissecting and rearranging pieces)	Congruent figures and similar triangles. Thales and Pythagoras Theorem. Relationship between three-dimensional shapes and their two-dimensional representation. Cartesian plane - ordered pairs, equations, intercepts, intersections, and gradient. Translation, reflection, and rotation.	structures 2. Are able to recognize the relationships between two-dimensional and three-dimensional figures. 3. Know how to use Pythagorean and Euclidean theorem (with not proof) to find length of a side
Data and Chance	**Standard 1:** To know how to recognize in different contexts the measurable nature of objects and phenomena, and know how to use measuring instruments (knowing how to locate the drive or the measuring instrument more suitable in a given context, knowing how to estimate a measure).	**Topic - Measures, Statistics and Probability** The student: 1. Is able to interprets data displays that could lead to misinterpretation (e.g., inappropriate grouping and misleading or distorted scales) 2. Is able to use data from experiments to predict chances of future outcomes	Reading and displaying data using tables, pictographs, bar graphs, pie charts and line graphs. Interpreting data sets (e.g., draw conclusions, make	The students: 1. Are able to compare different data from experiments to predict future outcomes 2. Are able to master procedures to

	Expected result: Use mathematics learned to treat amount of information in scientific, technological, economic and social (describe a phenomenon in quantitative terms, interpret a description of a phenomenon in quantitative terms with statistical tools or functions, use mathematical models to describe and interpret situations and phenomena).	3. Is able to use the chances of a particular outcome to solve problems	predictions, and estimate values between and beyond given data points). Judging, predicting, and determining the chances of possible outcomes.	solve statistical problems

Geometry topics selected by Turkey RET

Expected results		Learning content (topics, concepts, context and activities, inte-subject relations)		
Nucleus of the learning content	*Expected results at a curriculum level*	*Expected results divided into topics*	*Basic terms (divided into topics)*	*Context and activities*
Number	**Standard 1:** One knows all the fundamentals operations with exponents, **Standard 2:** One knows all the fundamentals operations with radicals **Standard 3:** One knows all the fundamentals operations with real numbers. **Expected result:** One knows to compute with exponents, radicals and real numbers and be able to solving problems using mathematical tools.	**Topic - Operations and Real Numbers** The student: 1. Knows and masters the specific contents of Mathematics (mathematical objects such numbers and their specific operations); 2. Knows and masters algorithms and procedures (most of all in arithmetic); 3. Is able to acquire progressively typical forms of mathematical thought (surmise, verify, warrant, define, generalize).	Computing, estimating with exponents. Exponents , scientific notation, radicals, using words, numbers, or models (including number lines) Computing with exponents and radicals.	The students: 1. Are able to understand the fundamentals of numerical operations 2. Are able to solving problems involving real numbers and their properties 3. Are able to solving problems involving radicals and their properties 4. Are able to solving problems

Algebra	**Standard 1:** One knows the basic ways of the factoring. **Standard 2:** One knows how to factorize binomials and trinomials using formulas. **Standard 3:** One knows how to simplify by using formulas. **Standard 4:** One knows the basic of the algebraic operations. **Standard 5:** One knows how to solve linear equations using formulas and graphing. **Standard 6:** One knows how to solve linear inequalities using formulas and graphing. **Expected result:** To learn solving problems using the tools of Mathematics (locate and connect relevant information, compare solution strategies, identify issues predicting patterns such as sequence of tasks, exposing the resolution procedure).	**Topic - Relations and Functions** The student: 1. Works with sums, products, and powers of expressions containing variables; 2. Is able to simplify or compare algebraic expressions; 3. Is able to model situations using factoring; 4. Evaluates formulas for given binomials or trinomials; 5. Evaluate slope of a line. 6. Evaluate the x and y intercepts. 7. Solves simple linear equations and inequalities, and simultaneous (two variables) equations.	Simplifying and evaluating the algebraic expressions. Simple linear equations and inequalities, and simultaneous (two variables) equations. Equivalent representations graphs, words, or equations. Modeling situations using expressions.	involving exponents The students: 1. Are able to understand how to solve expressions containing variables 2. Are able to model situations using expressions
Geometry	**Standard 1:** One knows the all the	**Topic - Figures and Bodies** The student: 1. Knows the properties of	Geometric properties of angles and lengths of the triangles.	The students: 1. Are able to

	properties related with the lengths of the triangle. **Standard 2:** One knows to construct geometry objects, described in the basic construction tasks. **Standard 3:** One knows to recognize shapes in space (identify relationships between shapes, images, or three-dimensional objects and conversely, represent a solid plan, seizing the properties of objects and their relative positions). **Standard 4:** One knows to construct the pattern of a line, polygons and circle **Standard 5:** One knows to symmetry and iteration on the coordinate plan. **Expected result:** To know different forms of representation and know how to go written, graphical, symbolic);	geometric shapes: triangles 2. Knows the relationships for median, altitude and angle bisector of a triangle. 3. Recognizes equality and congruent of triangle and their measurements 4. Recognizes similar triangles and recall their properties 5. Trigonometric relations on a right triangle. 6. Knows the relationships between two-dimensional and three-dimensional shapes 7. Knows how to measure, draw, and estimate the size of angles, the lengths of lines, areas, and volumes 8. Knows formulas for perimeters, circumferences, areas of circles, surface areas, and volumes 9. Knows the code of the symmetry and reflection 10. Knows pattern and fractal 11. Knows the way to take a symmetry.	Equal figures Congruent figures and similar triangles. Thales and Pythagoras Theorem. Trigonometric properties Relationship between three-dimensional shapes and their two-dimensional representation. Translation, symmetry, reflection, and rotation. Pattern and Fractal	construct and interpret different geometric structures 2. Are able to recognize the relationships between two-dimensional and three-dimensional figures. 3. Know how to use Pythagorean and Euclidean theorem (with not proof) to find length of a side 4. Are able to recognize the pattern 5. Are able to construct fractal
Data and Chance	**Standard 1:** To know how to recognize in different contexts the measurable nature of objects and phenomena, and know how to use	**Topic - Measures, Statistics and Probability** The student: 1. Is able to interprets data displays that could lead to misinterpretation (e.g., inappropriate grouping and	Reading and displaying data using tables, pictographs, bar graphs, pie charts and line graphs.	The students: 1. Are able to compare different data from experiments to

	measuring instruments (knowing how to locate the drive or the measuring instrument more suitable in a given context, knowing how to estimate a measure). **Expected result:** Use mathematics learned to treat amount of information in scientific, technological, economic and social (describe a phenomenon in quantitative terms, interpret a description of a phenomenon in quantitative terms with statistical tools or functions, use mathematical models to describe and interpret situations and phenomena);	misleading or distorted scales) 2. Is able to use data from experiments to predict chances of future outcomes 3. Is able to use the chances of a particular outcome to solve problems	Interpreting data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points). Judging, predicting, and determining the chances of possible outcomes.	predict future outcomes 2. Are able to master procedures to solve statistical problems

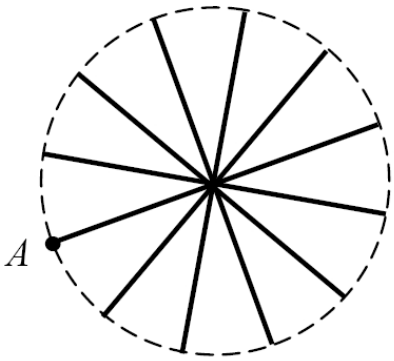
APPENDIX 2 – Proposed assessment schedules

Proposed assessment schedules Bulgaria

1. WINDMILL

In the past people used the wind power for windmills to grind grain, to get flour and oil, to pump water. Today, windmills are rather a tourist attraction. One of the most famous mills in Bulgaria is situated in Nesebar. It has 12 wooden blades with a joint center located at equal lengths from one another. All these things make it unique.

The drawing shows how its blades are situated.

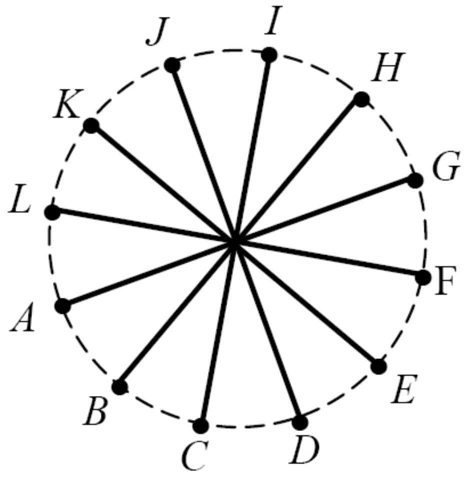


Question 1. What is the degree of two adjacent angles of the windmill in Nesebar?
Answer. ....$^\circ$

Question 2. The length of a windmill blade is 4 m. What is the LEAST integer number in meters the peak of a blade passes (e.g. A point) for a revolution?
Here, you can use the approximation π = 3,14.
Answer. .... m

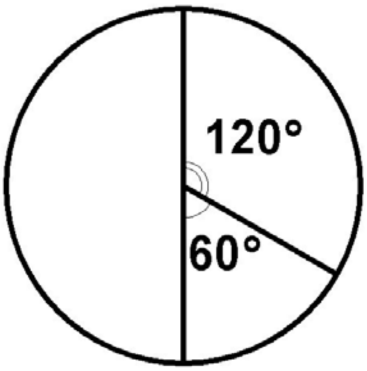
Question 3. The vertices of the blades are marked with capital letters. Using only the letters, write down, in the right-hand column of the table below, the figure that corresponds to the condition on the left.

Condition	Figure
A triangle, which is congruent to ΔAEG, is:	
Equilateral triangle with vertex in L is:	
The midperpendicular of the segment DF is:	
A right triangle with angle H equals to 30° is:	



2. CARNIVAL HATS

A coloured cardboard is in the shape of a circle whose center is point O and radius is 36cm. It is cut into three pieces out of which 3 carnival hats are made in the shape of a cone.

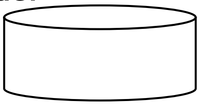
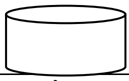
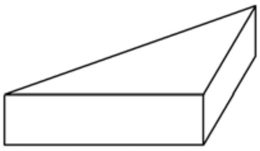
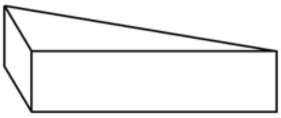
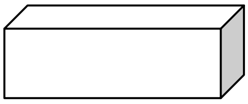


Question 1. What is the radius of each of the hats?

Question 2. Brims 5 cm wide have to be made for each of the hats. They are made out of red cardboard in the shape of a square. How many cm, at the least, should the side of the square be, in order to cut the brim for the biggest hat?

3. CAKES Zaharo

Zaharo sweet shop makes several types of standard-shaped cakes.

Types of cakes	Size
1.Straight circular cylinder 	Radius R = 18 cm Height 5 cm
2. Straight circular cylinder 	Radius r = 10 cm Height 4 cm
3. Straight prism whose base is a equilateral triangle 	Side a = 38 cm Height 5 cm
4. Straight prism whose base is a right-angled triangle 	The base has cathetus a = 30 cm and b = 40 cm, and hypotenuse c = 50 cm Height 5 cm
5. Straight prism whose base is a rectangle 	Width 40 cm and length 60 cm Height 5cm

The firm uses two types of package boxes.
- A straight prism whose base is a rectangle with width 42cm, length 62cm and height h = 8cm
- A straight prism whose base is a square with side 42cm and height is h = 8cm

Question 1. Which of the statements below is true and which is wrong?

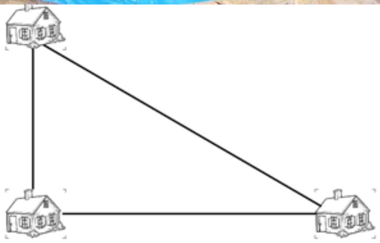
		True	Wrong
A.	All the types of cakes can be packed in boxes whose base is a rectangle		
B.	Two types of cakes cannot be packed in boxes whose base is a square.		
C.	Two cakes of the first type can be packed in boxes whose base is a rectangle		
D.	Three cakes of the second type can be packed in boxes whose base is a rectangle		

Question 2. The firm has to make a two-floor cake for a special occasion: the first floor is of a cake of the fourth type, and above it, on the second floor, is a cake in the shape of a cylinder. The base of the cylinder is fully situated onto the area of the second floor. How big, at the most, can the radius of the base of the cylindrical floor be?

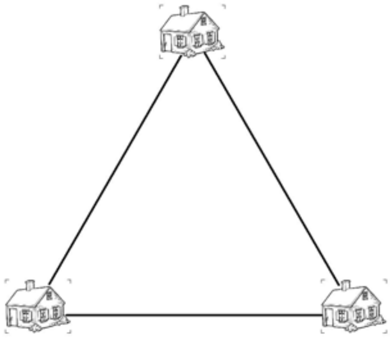
4. SWIMMING POOL

Three families are building their family houses in a gated complex. They can situate their houses in three ways:

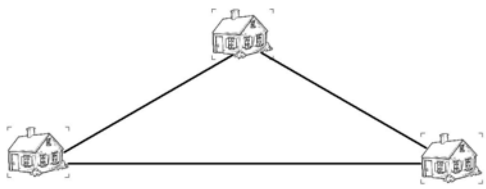
1. Each of the houses is situated at the vertices of a right-angled triangle with hypotenuse 600 m.



2. Each of the houses is situated at the vertices of a equilateral triangle with side 900m.



3. Each of the houses is situated at the vertices of an obtuse isosceles triangle with leg 400m and angle 120°.



The three families want to build a shared swimming pool, situated at equal distances from each house.

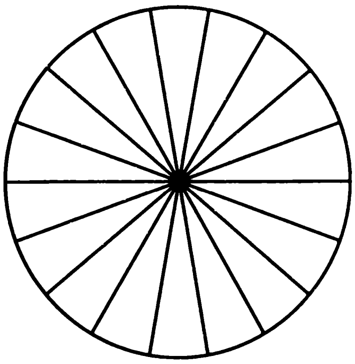
Question 1. Draw where the shared swimming pool should be in the three cases. What do you know about the point which is at the place of the swimming pool?

Question 2. At what distance from each house the swimming pool is in the different cases. (In your answer, if needed, give the approximate value in meters, with accuracy till whole unit)

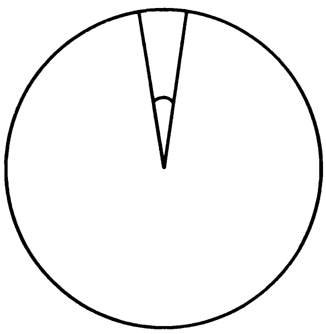
Question 3. In which of the three cases the distance from each of the houses to the shared swimming pool is the least?

4. **BICYCLES**

Question 1. A bicycle has 18 spokes. What degree is the angle of two adjacent spokes?

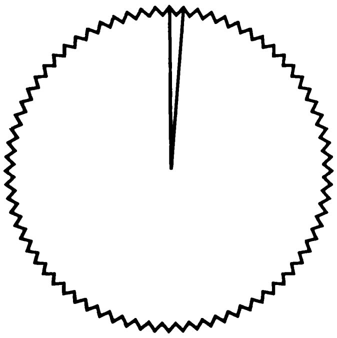


Question 2. How many spokes does a bicycle have if the angle of two adjacent spokes is 180?

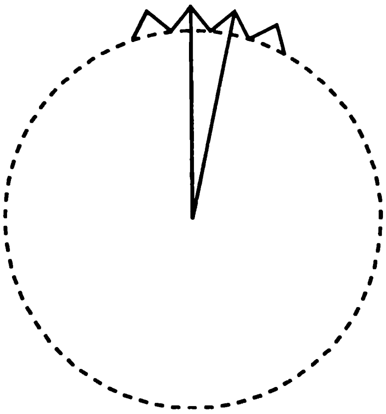


6. COGWHEEL

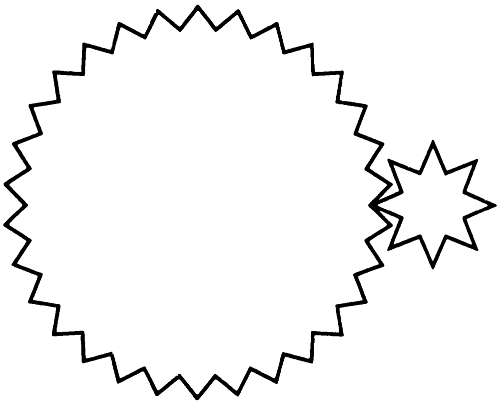
Question 1. A cogwheel has 72 cogs. How many degrees is the measure of the catenary of the circle, lying between two adjacent cogs?



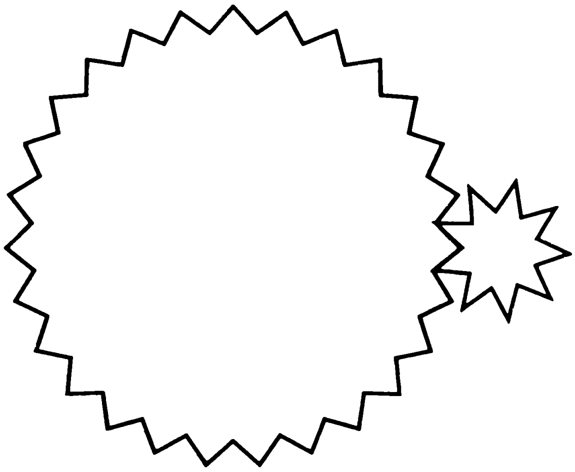
Question 2. How many cogs does a cogwheel have if the catenary between two adjacent cogs is 120?



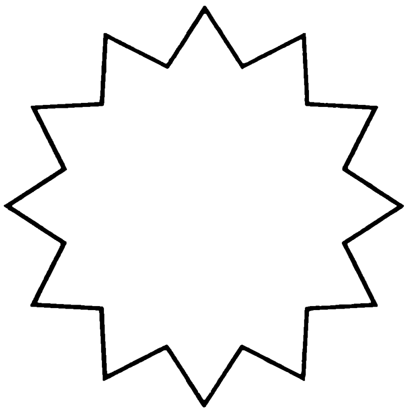
Question 3. How many revolutions per minute does a 32-cog cogwheel make if the other cogwheel, hitched to it, has 8 cogs and makes 12 revolutions per minute?



Question 4. The diameters of two adjacent cogwheels are in proportion 3:8. At what angle will the big cogwheel wind up if the small one makes one revolution?



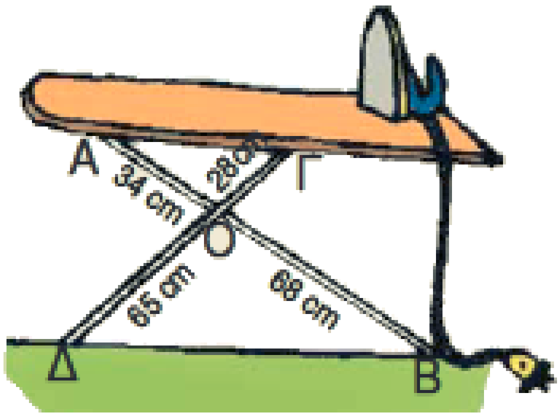
Question 5. A cogwheel has 12 cogs. How many cogs does the second cogwheel, hitched to it, have if for one revolution the first one winds up to 120o?



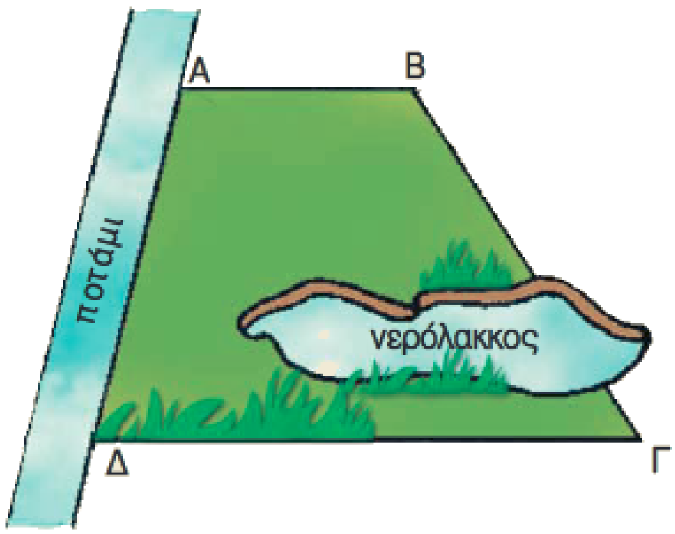
Proposed assessment schedules Greece

Common: Thales Theorem

John wanted to iron some shirts and mounted this ironing board, as shown in the figure. But he found out that the board is not flat. Where is the mistake John made?



1. Parallel lines
A farm is in the shape of a trapezium ABCD.
The owner wants to measure the perimeter to the fence, but the distance BΓ cannot be computed directly, as a pond formed by the recent rainfall has to be counted, as shown in the figure. How could you estimate this distance?



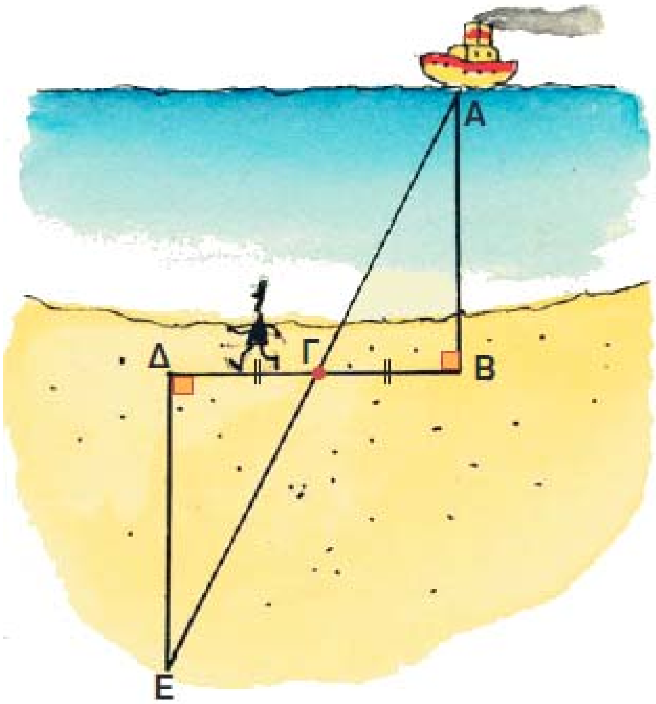
2. Similar triangles

Calculation of a distance from the shore to a ship.

If a ship is in position A and we stand at point B, then
• Starting from point B and walking on the beach perpendicular to AB we go a distance BΓ. At point Γ, we put a sign, (eg plant a stick) and continue going on the same line up

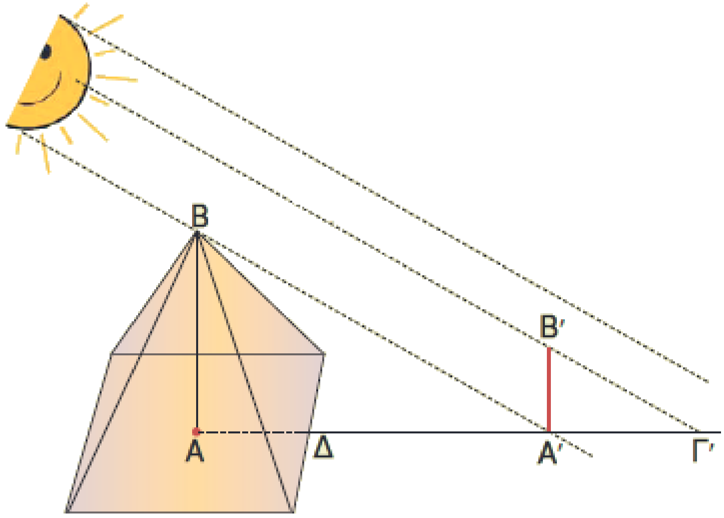
until the distance of ΓΔ = ΒΓ.
• At point Δ we put a sign, (eg a stone) and we turn and walk vertically, perpendicular to ΒΔ. We stop at a point E from which the points A and C appear to be on the same line.
The required distance AB is equal to the distance ΔΕ which we can measure.

This method is said to have been applied around 2,500 years ago by Thales of Miletus. How was Thales sure that AB = ΔΕ? Can you prove it? Find the five proposals that demonstrated Thales and note which of those used to calculate the distance from the ship to shore.



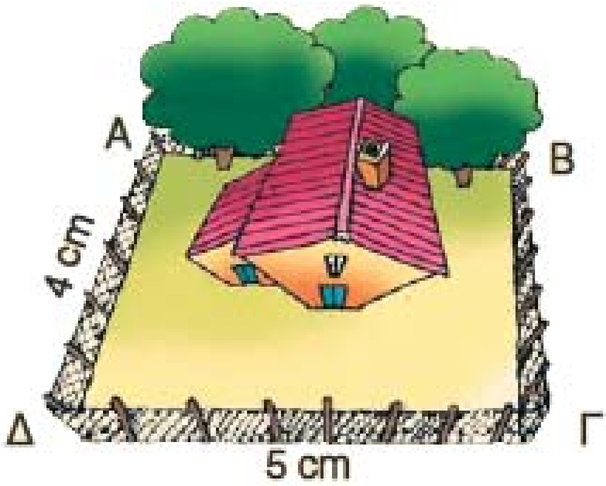
3. Similarity

The theory of similar shapes has been known since the mid-7th century BC.
With the help of this theory Thales of Miletus (624 to 547 BC), one of the seven sages of antiquity, was able to calculate the height of the great pyramid of Cheops by simply measuring the length of its shadow, winning the admiration of the King Egypt, Amasis.
We do not know the exact techniques used by Thales in this achievement. Plutarch, however, tells us that:
"Once Thales did set up a stick at the tip of the shadow cast by the pyramid, and thus having made two triangles by the sun’s rays, he showed that the ratio of the pyramid to the stick is the same as the ration of the respective shadows".
Diogenes Laertius, in fact, claims that Thales measured the shadow of the pyramid if the length of the rod was equal to the length of the shadow.
Can you explain how Thales estimated the final height of the pyramid knowing that he could measure the length of the side of the square base of the pyramid of shadow ΔΑ’?



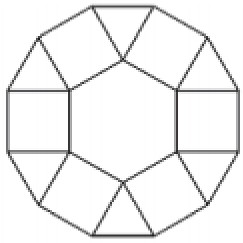
4. Homothety

This figure shows an aerial photograph of a farm with a rectangular shape which is fenced by barbed wire with a length of 270m.
Compute the actual dimensions of the farm.
On what scale was the aerial photography taken?



Proposed assessment schedules Italy[5]

1. The images, here below, represent a ground floor of an ancient Roman house and its geometrical schematization.



The pattern, corresponding to a dodecagon, consists of a regular hexagon inside, six squares and six equal equilateral triangles.
Indicates whether the following assertions are true or false.

		True	False
a.	The area of the hexagon is half the area of the dodecagon	❑	❑
b.	The area of each triangle is one-sixth of the hexagon	❑	❑
c.	The area of a square is twice the area of a triangle	❑	❑
d.	The perimeter of the dodecagon is double the perimeter of the hexagon	❑	❑

2. We want to paint a separation wall between two gardens of adjacent houses. The wall, long 5 m, with a thickness of 0.2 m and a height of 1 m, supports one of the side panels on the walls of the houses, as shown.

How much it measures the surface to be painted?

[5] From http://www.studenti.it/files/pdf/20100618/fascicolo_matematica2010.pdf

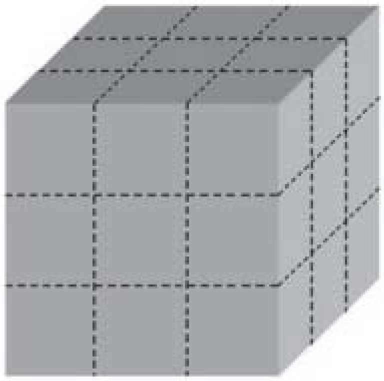
☐ A. 10,4 m^2

☐ B. 11,2 m^2

☐ C. 11,4 m^2

☐ D. 12,4 m^2

3. The surface of the wood cube in the picture is completely painted. Then the cube is sawn along the dotted lines. We get so many different cubes, some of those have no painted face, the others one or more faces painted.



Now, please, complete the following table.

No. faces painted	No. cubes
0	
1	
2	12
3	

4. A peeled pot, of 4 kg, is 11 cm high and has a base of 6 cm in diameter. What is the volume of the pot?

☐ A About 100 cm^3

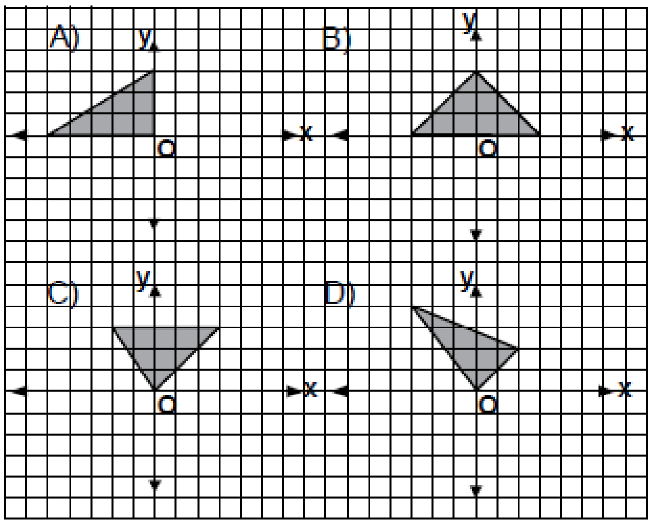
☐ B About 200 cm^3

☐ C About 300 cm^3

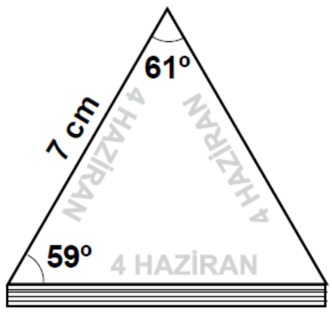
☐ D About 400 cm^3

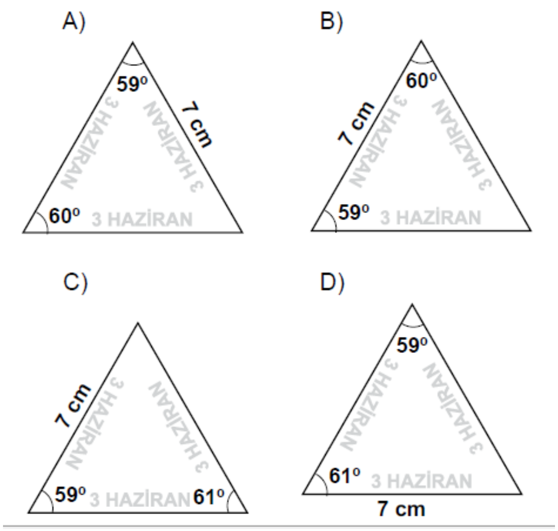
Proposed assessment schedules Turkey[6]

1. Which of the following is same as itself when it is reflected on x – axis then rotating 180 ^{0} about the origin.



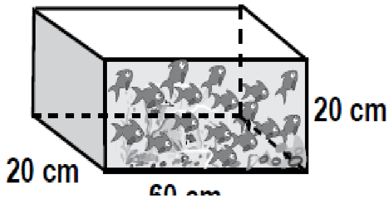
2. The page of the calendar is a triangle which is given. Which of the following will belong to same calendar?

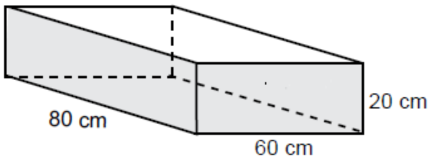


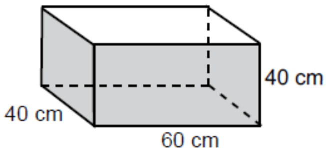


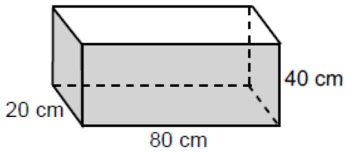
[6] From : http://www.meb.gov.tr/duyurular/duyurular2011/EGITEK/SBS2011/sbs2011_8a.pdf

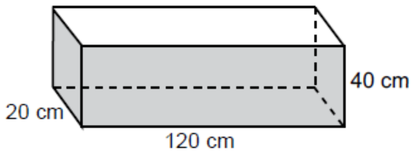
3. A worker man made a rectangle prism aquarium which is given on the diagram. He wants to make a new one which the volume is 4 times the old one. Which of the following can't be an example for the new one?



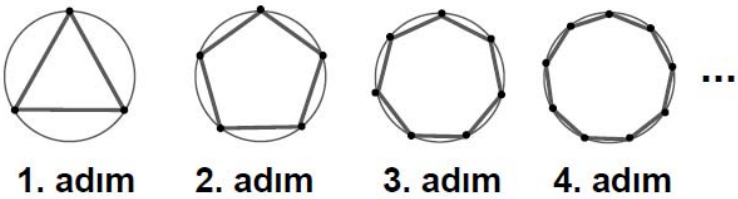
A)


B)


C)


D)


4. The pattern is given. If it continues what is the number of sides of the inscribed polygon on the 19th step.



A) 24 B) 33 C) 39 D) 42

5. A person was making a research about "which color of car is sold a lot". He found that white color was the first. By looking at which knowledge he would get that result ?

A) Median B) Biggest Value C) Mean D) Range

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